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AN INVESTIGATION OF THE QUEUE DISCIPLINE AT A RETAIL STORE COUNTER

A THESIS

Presented to

the Faculty of the Graduate Division

by

John William Windle, Jr.

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Industrial Engineering

Georgia Institute of Technology

December, 1957

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AN INVESTIGATION OF THE QUEUE DISCIPLINE
AT A RETAIL STORE COUNTER

APPROVED:

Date Approved by Chairman: Dec 16, 1957

ACKNOWLEDGEMENTS

The writer would like to express his sincerest appreciation to all those who have assisted in making this work possible. Doctors Joesph J. Moder, Robert N. Lehrer, and M. Carr Payne are accorded a particular thanks for their direction and assistance. A word of appreciation is due Rich's, Inc. and its personnel for their cooperation and assistance during the collection of the data for the study. This writer is also indebted to his wife, whose encouragement and patience helped make this study possible.

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SUMMARY

The object of this study was to investigate the feasibility of the application of the queuing theory to a retail sales point, with particular emphasis on the queue discipline. The specific objectives of the study were as follows:

1. To determine reasons for customers' leaving sales counter queues
2. To determine the nature of departures from a first-come, first-serve sales policy, and their effect on the customer waiting time distribution
3. The development of a method of applying the queuing theory in the determination of the optimum assignment of sales clerks

The data were collected at Rich's, Inc. by personal interviews and time lapse cameras. The study was conducted at the lady's belt counter for a period of two eight-hour days.

The results drawn from this study are as follows:

1. For both days ninety-six per cent of the customers leave for reasons other than the length of the waiting line
2. There is a small departure from a first-come, first-serve sales policy. As the size of the queue increases so

does the variation in the order of service

3. A mathematical model of a retail sales point requires a special approach due to the effect of the queue discipline on the expected waiting time

It is recommended that further study be conducted on this subject in the direction of

1. Characterizing the queue discipline
2. Determining the validity of the suggested economic application
3. Determining if more sales personnel would attract the customers that looked without being served

CHAPTER 1

INTRODUCTION

Since the beginning of World War II, the use of the laws of probability has brought new insight to the behavior of many physical systems. One of the techniques which has been shown to be a practical and valuable tool is the queuing theory; or, as it is sometimes called, "the theory of waiting lines."

The development of the queuing theory led to mathematical models which would predict the behavior of a system that attempted to provide service to randomly arising demands for service.

Although the most numerous applications have been made since 1950, it was in 1908 when the first published article on this theory appeared. The author, A. K. Erland, studied the problem of telephone traffic congestion in the pioneer article, "Use of Waiting Line Theory in the Danish Telephone (1)[1]." This initial interest in the field was finally reflected, and in 1927 Molina (2) published his work on the telephone. From 1928 until 1940 such men as Pollaczek (3), Khintchine (4), and Crommelin (5) contributed greatly to the development of the queuing theory.

However, it was not until the beginning of World War II that

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[1]Numbers in parentheses refer to references listed in Bibliography.

the queuing theory got its biggest impetus. This was primarily due to the increase of interest in Operations Research, for which the queuing theory became one of the many techniques. The types of work done during this period are illustrated by R. Kronig's (6) "On Time Losses in Machinery Undergoing Interruptions" and W. F. Weir's (7) "Figuring Most Economical Assignment."

In the social and manufacturing fields almost every operation may be described as a "gate". A certain service time is required for each unit of material or person arriving at the gate before he or it can be passed on and the next accepted. If the demand or input to the gate exceeds the capacity of service for a short period of time, the people or material will be kept waiting. Usually the person or item forming the queue arrives at the gate, but sometimes the gate moves (i.e. a waitress in a restaurant). This waiting of material and people can become costly; and for this reason, the interest in this field has increased.

M. G. Melden's (8) definition of the queuing theory can be rephrased as, theories used to determine the probability of bottlenecks occurring and to show the effect which changes of service would have on the size of the resulting line. His definition implies that the queue may consist of persons or items and does not necessarily mean that the queue will take the form of a physical line. This broad approach to the waiting line problem could be more generally defined as an approach to the study of congestion.

The successful application of queuing theory are shown in such studies as:

1. Telephone congestion (9)
2. Landing of aircraft (10)
3. Loading and unloading of ships (11)
4. Scheduling of patients in a clinic (12)
5. Railroad classification yards (13)

More applications are cited in an extensive bibliography on the queuing theory by Vera Riley (14).

According to D. G. Kendal (3) each queuing process has four characteristics. They are:

1. Input (the way customers arrive at the point of service)
2. Queue discipline (the manner in which customers queue up for service
3. Service mechanism (the mode of service)
4. Output (the departing customers' method of leaving the system

These characteristics follow very closely those of Marshall (15) who listed the following:

1. Customers
2. Gate or service point
3. An input process
4. Queue discipline
5. Service mechanism

Of the many published applications of the queuing theory, none has concerned itself with the retail sales point. It is true that theoretical studies have been made in which the retail sales counter was given as an example, but up to now no application of the queuing theory to retail stores has been published. The lack of information, as well as the difficulty of obtaining data necessary to describe the operation may be one reason such a study has been delayed. Nevertheless, a department store counter provides an excellent location for a study of the queuing theory, for basically it has all the characteristics of the queuing problem. A brief description of these is as follows:

1. Customer--the person that arrives at the counter for service
2. Gate or service point--the counter that the customer approaches for service
3. Input process--the random frequency pattern at which customers arrive for service and enter the queue
4. Queue discipline--each customer is governed by a moral code, queue discipline, which is the manner in which customers react to the order of service and the length of the service time
5. Sales mechanism--each customer requires a certain time for service, after which he leaves the counter
6. Output--at a department counter the output is of little interest as the customers' next destination is irrelevant.

A study of a department store counter by the application of the queuing theory would bring results with which an attempt could be made to control the length of the waiting line and the sales clerk's idle time.

These two objectives are diametrically opposed, for a decrease in one would cause an increase in the other or vice versa providing the queue is of finite length and small in size. This suggests that there exists an optimum economic condition where a balance might be brought about between the cost associated with loss of sales due to lengthy waiting lines and the cost of the sales clerk's idle time.

A study by D. Y. Barrer (16) of a waiting line characterized by impatient customers and indifferent sales clerks was similar to this study. He assumed a Poisson type input and output. If x is the number of customer arrivals or customers served in elapsed time, t , then a Poisson type input or output is one in which the probability of x is given by the Poisson probability functions as follows:

$$p\ (x) = \frac{e^{-\lambda t}\ (\ t)^{x}}{x!}$$

Frequently in queuing studies, one wishes to refer to the probability function of time intervals between customer arrivals or customers served. This function is of the negative exponential type given below providing that the input or output is of the Poisson type as stated previously.

$$p\ (t) = \frac{1}{\lambda}\ e^{-\lambda t}$$

Throughout this thesis, the Poisson type or negative exponential type input or output implies the above functions. To specify a queue completely, the distribution function for the arrival must be given. The type of queue discipline, whether first-come, first-serve; priority;

or random selection for service must be determined. The number of channels (sales clerks) and the service time distribution are also needed to completely specify a queue.

Once the information is obtained and each characteristic adequately described, it will be possible to derive a mathematical model that will represent the physical system. With the aid of this model the flow of customers can be simulated through the sales point, permitting an analytical study of all variables that affect the queue.

CHAPTER II

THE PROBLEM

The object of this study is to investigate the feasibility of the application of the queuing theory to department store retailing and to describe the requirements of a mathematical model to represent this physical system.[1] This particular investigation was concerned with the problems that arise as a result of the queue discipline around a retail sales point.

Since the purpose of this thesis is to provide ground work for further mathematical investigations, it will touch only the broad aspects of this problem and will not concern itself with minute details. From this study it is hoped that promising areas will come for future research.

The specific objectives of the study reported in this thesis are as follows:

1. To determine reasons for customers' leaving sales counter queues.
2. To determine the nature of departures from a first-come, first-serve sale policy, and their effect on the customer waiting time distribution.
3. The development of a method of applying the queuing theory

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[1]This general objective is being carried out by a series of studies, of which this is one, at the Georgia Institute of Technology.

in the determination of the optimum assignment of sales clerks.

These specific objectives can not be transposed into hypotheses and tested, for this thesis is one of estimation and is intended as a guide for future development of this topic.

An attempt shall be made to determine if the sales policy varies from first-come, first-serve, as the degree of variation from such a policy affects the waiting time. If the sales policy approaches random service, it is possible to keep an individual waiting for a long period of time, thereby affecting the importance of the physical queue.

The waiting time of a customer will be analyzed to see if some simple relationship exists between it and the other properties of a retail sales point. Another aspect that will be examined is the reason why customers leave the counters. If a sufficient number of customers leave because of the length of the waiting line, then additional sales clerks must be added to correct this situation. If, however, there are ample service facilities and the customers continue to leave, the answer to the problem lies elsewhere.

To present an economic use of the queuing theory to a retail sales point an additional step must be taken so that the results of the study can be used for the optimum assignment of sales personnel. Such a use of the theory shall be presented, but more study of the subject will be necessary to establish the validity of the approach.

CHAPTER III

EXPERIMENTAL PROCEDURE

An experimental investigation of a waiting line at a department store counter was made by the time lapse photography and personal interviews. The data were collected jointly with W. A. Gresham (17) whose thesis concerns itself with the arrival and service distribution of a retail sales point.

Selection of Location.--In the preliminary study a search was made in Atlanta for a retail sales point which was suitable for this experiment. The way in which the data was to be collected restricted the type of sales points that could be investigated. A counter had to be found that allowed full coverage of its area by the cameras. This meant that the counter had to be fully enclosed with the sales personnel confined within. The preliminary search ended with the selection of three counters that were considered suitable for the study. All three were in Rich's Department Store and were as follows:

1. Lady's belt counter
2. Lady's blouse counter
3. Fine jewelry counter

Since the experiment was to be conducted on a typical counter, the fine jewelry counter was discarded, as it sold a low-volume, high-priced merchandise. Although the lady's blouse is a high-volume, low-priced article, the problem of photographing the counter was more difficult than that of the lady's belt counter. Thus the selection of the lady's belt counter at Rich's was made, as it offered the best location to conduct the experiment. A layout of the counter and the ranges of the cameras can be seen in Figure 1. Also pictures taken at the location of the cameras are shown in Figures 2 and 3.

To insure that the volume of business was large enough to get sufficient data, the belt counter was surveyed at different times on Tuesday and Saturday. These checks revealed that the counter was suitable for the experiment. To be certain that the cameras would encompass the entire field, a trial was made; and the results obtained were satisfactory.

Equipment Used.--The data were collected on film with two sixteen millimeter Keystone A-9 cameras with thirteen millimeter wide angle lenses. These electrically driven cameras had been modified to run at twenty-five frames per minute. Each camera held one hundred feet of film, enabling the camera to run two hours and fifty minutes without being replaced. It was necessary then to change the film three times per day. To prevent gaps in the data, the starting time of the cameras was staggered. Two synchronized thirteen inch clocks were placed on the counter so that the film from the two cameras could be coordinated.

Selection of Time.--The experiment was to consist of two eight-hour days chosen so that the activity of traffic customers at the counter would be normal one day and above normal on the second day.

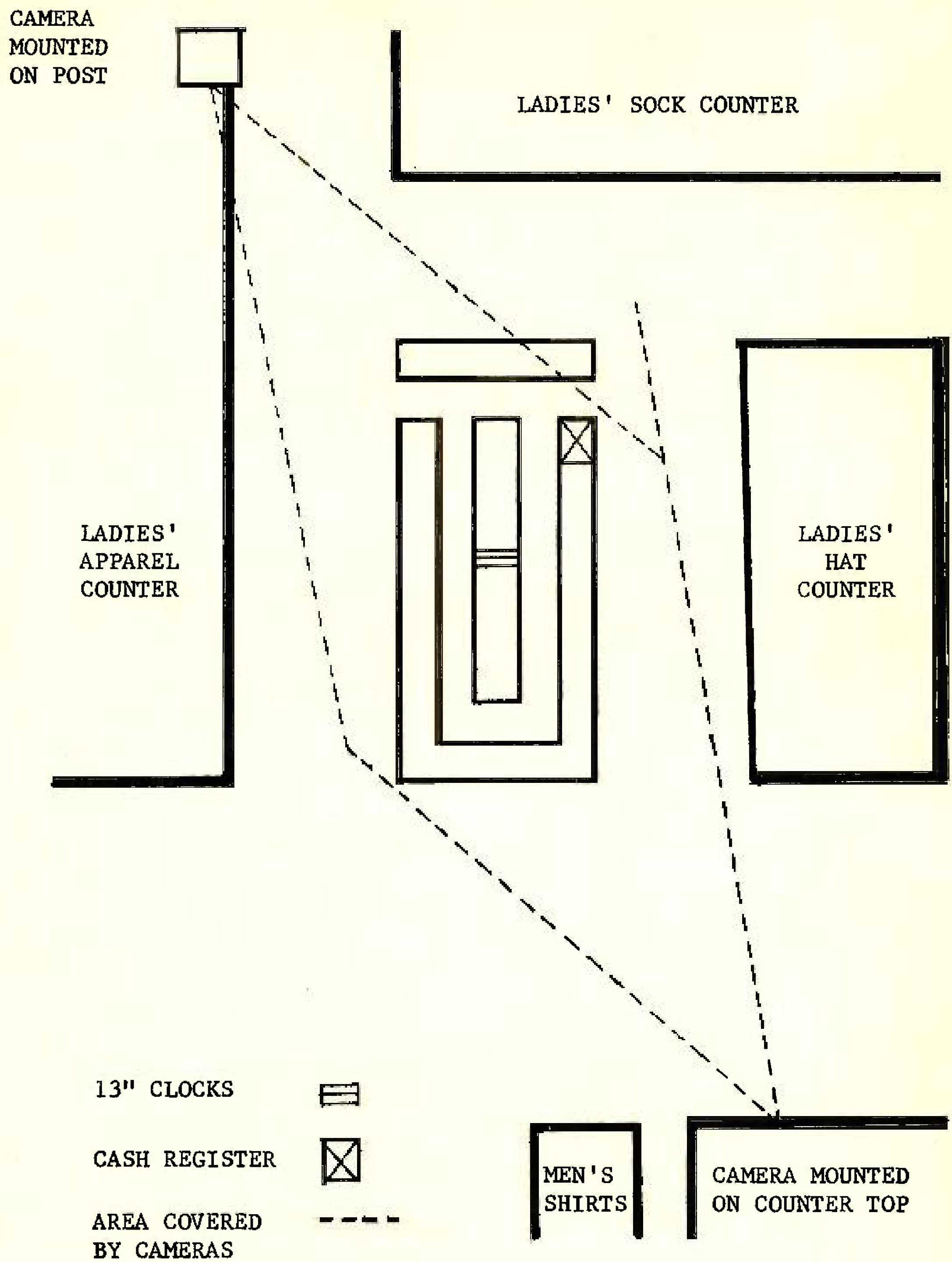

FIGURE 1 LAYOUT AND CAMERA COVERAGE OF BELT COUNTER

FIGURE 2 BELT COUNTER AS SEEN FROM POST MOUNTED CAMERA

FIGURE 3 BELT COUNTER AS SEEN FROM COUNTER MOUNTED CAMERA

By looking at their previous records, the management of Rich's recommended Tuesdays and Saturdays. The experiment was conducted on July 23 and 27, 1957.

Subjects.--All persons that were affected by the experiment were told why the study was being made. This was done in hopes of attaining normal working conditions. To prevent unnatural behavior of the customers, the cameras were concealed as much as possible by painting the camera mounts and selecting concealed locations. The attempt at concealment brought good results as no customer noticed the cameras until she had left the counter. This fact was confirmed in the personal interview.

Interviewing.--In order to supplement the data from the films, each person that did not purchase an item was to be interviewed. However, due to the length of time required to interview one person and the number of personnel available to conduct the interview, only those customers who stayed longer than two minutes at the counter and those who took up the sales clerk's time without buying were interviewed. The interview form is shown in figure 15 of the Appendix.

In the first part of the interview an attempt was made to ascertain why customers approached a counter. If a customer was looking without any intent of buying, then the interview was discontinued. If, however, there was some indication that there might have been a sale, the interview was continued. The second section of the inquiry tried to determine the reasons why the customers had left the counter without buying. Only those customers who left the counter because of the length of the waiting line were asked the third part of the interview in an attempt to see if the queue size had resulted in lost sales. The last section of the interview was designed to determine the belt price range desired by the customer.

Film Analysis.--After the films were developed, they were analyzed frame by frame, and the arrival and departure of each customer noted. This information was transferred to a master chart which showed the arrival, waiting, and service time of each customer; the number of people waiting to be served; the order in which they were served; and the termination of each transaction, ending either in a sale or no sale. A sample of the technique used to record this data is given in Figure 4. As seen in the figure (using as a scale 25.0 millimeters equal one minute), customer 505 arrived at the queue at 4:07:39 (hours:minutes:seconds). As there was no one at the counter when she arrived, she was serviced in turn at 4:07:49. Her waiting time was then the difference between her arrival time and the time her service was begun, in this case, ten seconds. Customer 505 left without purchasing at 4:11:01. While customer 505 was waiting to be served, customer 506 arrived at 4:07:49. An instant later when customer 505 was serviced, customer 506 dropped to the number one waiting level. The number one waiting level indicates the number of people waiting to be serviced as one, and the level shows the person should be serviced next. This same procedure was used throughout the master chart.

After the data for both days were recorded on the master charts, data sheets were made up. These sheets put in tabular form the information on the master chart. By subtracting the arrival time from the time the customer left the counter, the total time at the counter was obtained. A sample data sheet is given in the Appendix, figure 16. The data sheets are self-explanatory except for the one column which is entitled "Sale or No Sale." If there was a sale, it would be signified by the letter "S"; if there was no sale the personal interview results were put under this column. The interview form was coded, enabling an answer to be signified by a numeral and a letter. In the case of customer 505, since she did not purchase an article, the answer to her interview was 1-D. The explanation of this interview code is given in the Appendix, Table 9, which in this case meant that she was definitely going to purchase a belt, but that the color she wanted was not available.

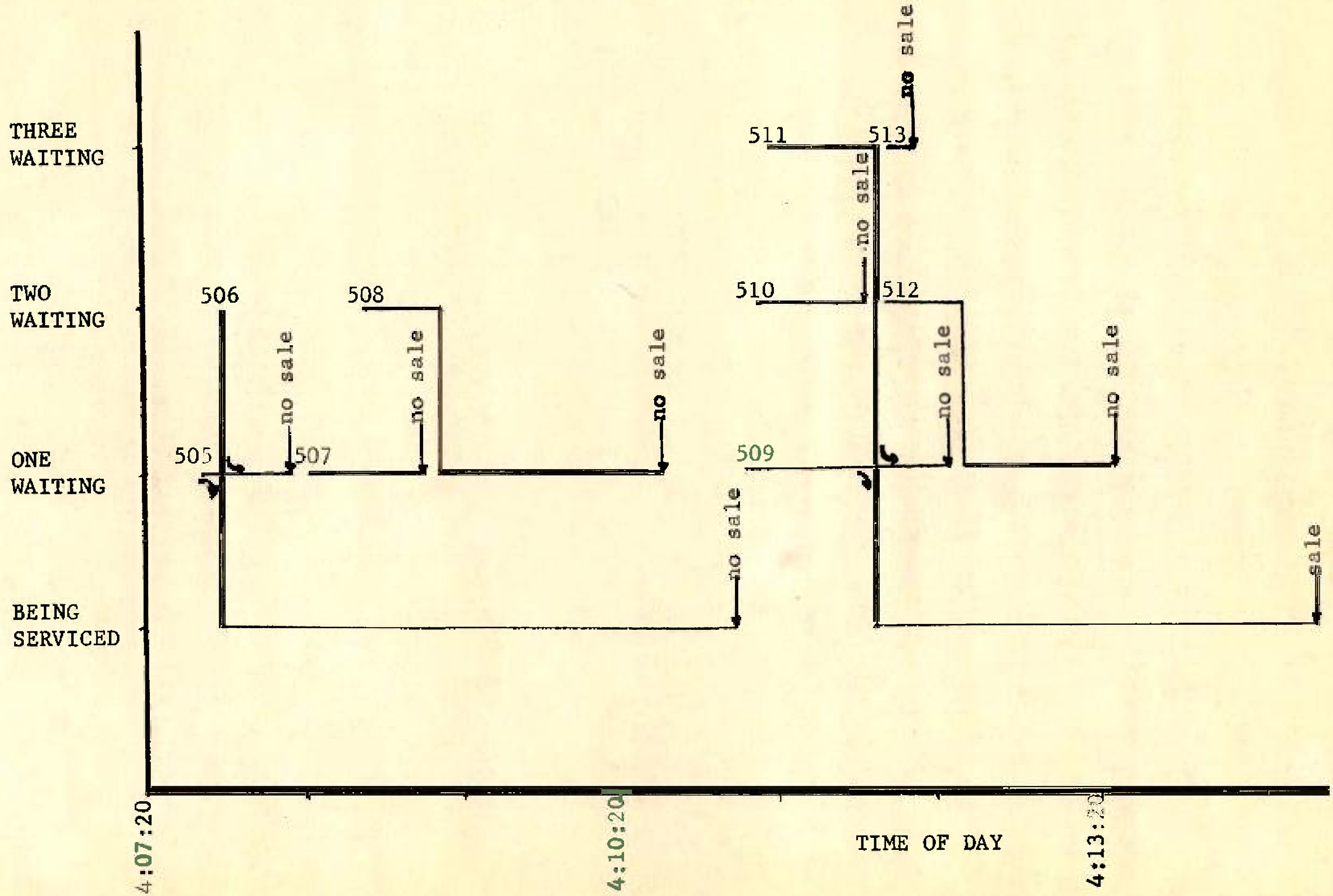

FIGURE 4 SAMPLE OF MASTER CHART FOR TUESDAY

CHAPTER IV

ANALYSIS OF DATA AND RESULTS

The analysis of the data will be divided into four sections. The sales policy is discussed in the first section. The second concerns itself with why people leave a counter. The third part of the analysis is a discussion of the waiting time distribution at a retail sales point. To give an economic approach of the application of the queuing theory is the goal of the last section.

The Sales Policy.--The sales policy of most retail department stores is based on a first-come, first-serve principle. This can only happen, however, under an ideal situation, for as the size of the queue increases, the difficulty of maintaining the correct order increases. Another cause for the variation from a first-come, first serve sales policy is that the experienced sales clerk has some indication of which customers might buy and which are just browsing. It is only natural to choose the one that might purchase for fear that she might get impatient and leave.

To collect the data necessary for an analysis of the sales policy, the customer's order of service was observed. If a person was serviced ahead of another person who had been in the queue longer, she was given a value of plus one. If she was serviced after someone who arrived at the queue after she did, she was given a minus one.

Thus a person receiving a plus three meant that she had stepped ahead of three people, while a person that received a minus three had three customers--who arrived at the counter after she had--serviced before her. From Table 1, the order of service can be seen for each day. Considering the absolute values for the order of service, the data were regrouped as seen in Table 2. With this data, the fraction of total occurrences for each order of service was determined. The results are shown graphically in Figures 5 and 6. The data were then fitted with a ninety-five per cent confidence interval. A person may interpret this confidence interval by saying that one is ninety-five per cent confident that the fraction of total customers for Tuesday that are serviced in turn (correct order) will be between 0.72 and 0.85.

It is easily understood why, as the size of the queue increases, the sales clerks become more confused as to the order in which they should serve customers. To investigate this point further, data were tabulated for both days on the size of the queue versus the order served and are shown by Table 3. The correlation coefficient for each day was determined by the method of Pearson's Product Moment. A regression line was determined and a standard error of estimate obtained. The following are the equations that resulted:

$$\text{Tuesday} \quad y = .10 + .18x$$

$$\text{Saturday} \quad y = .15 + .10x$$

The results show that for both days the line had a positive slope, indicating that as the size of the queue increased so did the order of

TABLE 1

THE FREQUENCY OF OCCURRENCE OF THE ORDER IN WHICH CUSTOMERS ARE SERVED

ORDER OF CUSTOMER SERVICE	TEST DAYS	
	TUESDAY	SATURDAY
+ 4	0	0
+ 3	2	1
+ 2	4	3
+ 1	24	17
0	137	174
- 1	8	5
- 2	0	1
- 3	0	0
- 4	0	1

NOTE:

+ INDICATES A CUSTOMER WAS SERVED AHEAD OF OTHER CUSTOMERS WHO ARRIVED AT THE COUNTER FIRST.

- INDICATES A CUSTOMER WAS SERVED AFTER OTHER CUSTOMERS WHO ARRIVED AT THE COUNTER LATER.

TABLE 2

ABSOLUTE SALES POLICY *

ORDER OF SERVICE	TUESDAY OCCURRENCES	SATURDAY OCCURRENCES
0	137	174
1	32	22
2	4	4
3	2	1
4	0	1
TOTAL	175	202

* THIS IS THE ABSOLUTE SALES POLICY WITH NO REGARD TO THE SIGN OF THE ORDER OF SERVICE.

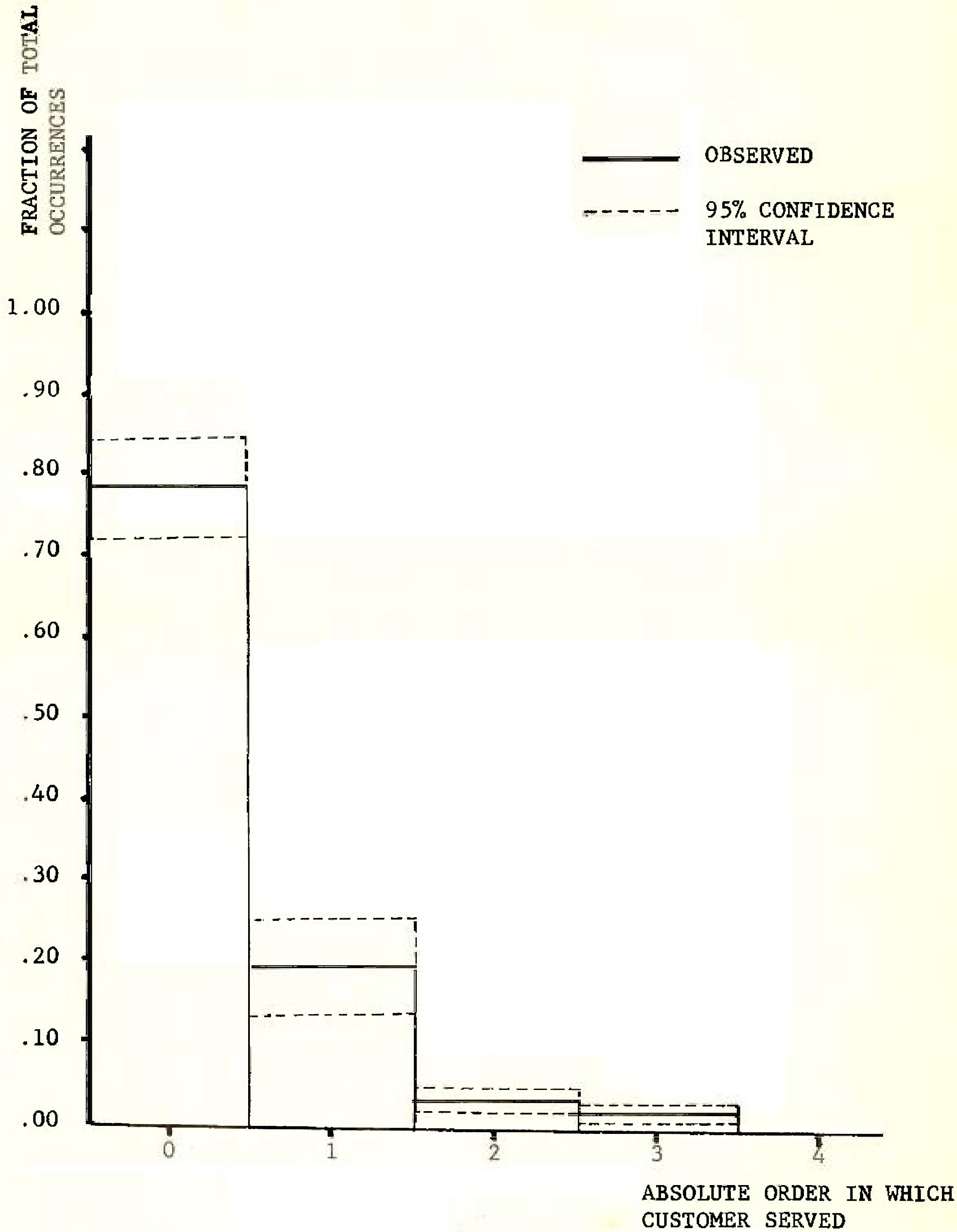

FIGURE 5 TUESDAY'S SALES POLICY

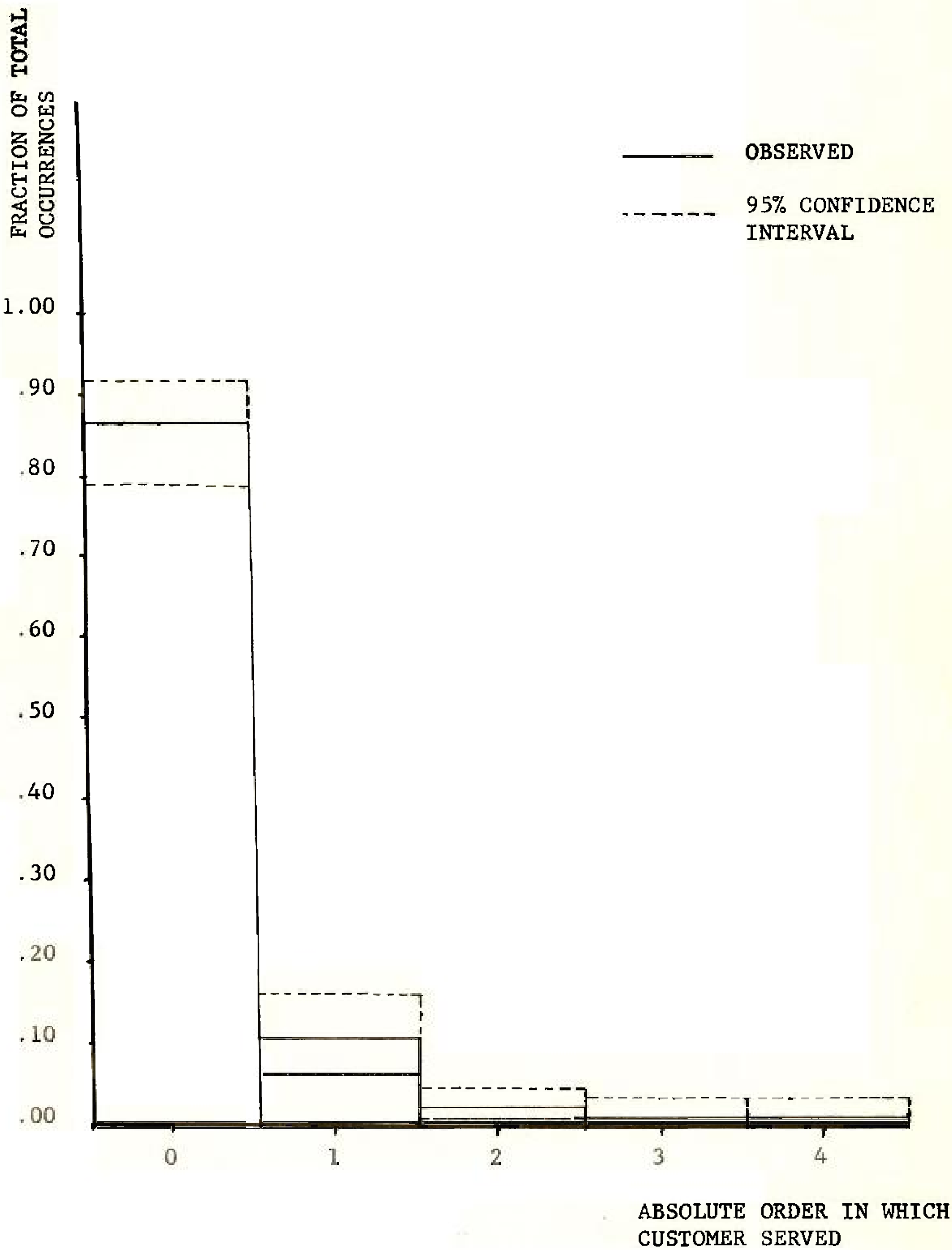

FIGURE 6 SATURDAY'S SALES POLICY

TABLE 3

QUEUE SIZE VERSUS ORDER SERVED

TUESDAY

SIZE OF QUEUE WHEN SERVED

ABSOLUTE		0	1	2	3	4	5	TOTAL
				OCCURRENCES				
ORDER	0	82	36	13	6	1	0	139
IN WHICH	1	3	18	7	2	0	0	30
CUSTOMER	2	0	0	3	1	0	0	4
IS SERVED	3	0	0	0	2	0	0	2

CORRELATION COEFFICIENT .396

STANDARD ERROR OF ESTIMATE 5.4%

SATURDAY

SIZE OF QUEUE WHEN SERVED

		0	1	2	3	4	5	TOTAL
				OCCURENCES				
ABSOLUTE	0	98	42	19	6	2	1	168
ORDER	1	5	19	2	1	0	0	27
IN WHICH	2	2	0	3	0	0	0	5
CUSTOMER	3	0	0	0	1	0	0	1
IS SERVED	4	0	0	1	0	0	0	1

CORRELATION COEFFICIENT .195

STANDARD ERROR OF ESTIMATE 5.8%

service.

One of the limitations of this method of obtaining the data could have affected the analysis. Since the camera took a picture every 1/25 of a minute, it is possible that between frames the customer that was next to be serviced told the sales clerk to wait on some one else. This limitation of the cameras is not important as in most cases it was obvious whom the sales clerk had approached and intended to serve.

Why Customers left the Queue.--The results of the personal interviews were tabulated for each day and the results shown in Table 4. The results indicate that people left for the same reasons on both days.

Of particular significance is the fact that less than four per cent left each day because of the length of the waiting line. This means that the length of the waiting line had little effect on those customers that entered the queue. This is no indication of the number of people that were discouraged and did not enter because of the size of the queue. However, this last statement is not particularily applicable to this counter for regardless of the size of the queue, a customer could always see the merchandise. The major difference between a retail store's queuing problem and other queuing problems is that most retail service points employ some form of self service which does not require constant servicing. Only when the customer decides that she is going to purchase an item does the size of the queue become important. There are numerous people that approach a counter, look, and leave. The problem is raised as to whether or not these are customers or just curious people. In most

TABLE 4

RESULT OF PERSONAL INTERVIEWS

	TUESDAY			SATURDAY			
	% of Total Interview	95% CONFIDENCE Interval Low	High	% of Total Interview	95% CONFIDENCE Interval Low	High	WEIGHTED AVERAGE % for both days
WHY CUSTOMERS APPROACH COUNTER							
1. Definitely going to buy a belt	25.9	17.5	35.0	41.5	27.5	52.0	32.7
2. Perhaps might buy belt	37.0	27.0	47.0	18.8	9.0	33.0	33.3
3. Just Browsing	37.1	27.0	47.0	39.6	26.0	55.0	38.1
WHY CUSTOMERS LEFT COUNTER							
1. Too many people waiting	3.6	1.0	10.0	1.8	0.5	12.0	3.2
2. Did not have size	12.9	8.0	23.0	18.8	9.0	33.0	15.0
3. Did not have style	52.3	42.5	62.5	49.0	39.0	64.0	51.3
4. Did not have color	19.3	12.5	29.0	22.6	12.5	32.0	20.5
5. Other reasons	11.9	6.0	20.0	7.5	2.0	15.0	10.8
PRICE RANGE OF CUSTOMERS							
1. $1-3	50.0	40.0	60.0	41.5	27.5	52.0	47.2
2. $3-5	28.0	20.0	39.0	34.0	21.0	49.0	30.3
3. $5 and up	6.0	2.0	14.0	15.1	7.0	28.0	6.6
4. Didn't notice price	16.0	9.0	26.0	9.4	3.0	22.0	12.9

applications of the queuing theory today, only those people that require service are considered customers, and there is not a need to depart from that idea at a department store counter.

From the results obtained from the interviews it was surprising to see the number of customers that were definitely going to purchase a belt. On the two days that the customers were interviewed greater than thirty-two per cent said that they were definitely planning to buy a belt. It is true that the way the person was approached by the interviewer could have brought this result. Yet a closer look at the reason why these people failed to buy revealed that seventy-two per cent of these left because they were unable to find the style or color that they desired. This is no reflection on the inventory of the store as this study was made at a time when the merchandise was affected by the changing of the seasons.

Of the five people that left because of the length of the waiting line, not one of them said that they would return to the counter the same day. Even though the sample size of part three of the interview was too small to draw any definite conclusions, the results indicate that there was a definite loss of potential sales. This loss, however, would not necessarily justify the added expense of additional sales personnel; for assuming that the five people purchased a belt, this would not equal the wages of the sales clerk.

Part four of the interview revealed that in most cases, the prices of the belts were in the customer's price range. When asked what their price range was, greater than forty-seven per cent of the customers replied that their price range was from one to three dollars. On Tuesday, when there was a remnant sale in the store, fifty per cent of the customers were interested in the lower price range. This result, plus the fact that on Tuesday only thirty-two per cent of those people served purchased indicates that on Tuesday people were looking for a bargain. This compares with Saturday when forty-two per cent purchased a belt.

Waiting Time Distribution.--Unlike the arrival and service distributions, the waiting times are not independent. From the data given in Tables 7 and 8 in the Appendix, Figures 7 and 8 were made to show the time of day versus the arrival rate, servicing time, and the waiting time averaged for thirty-minute periods. If the waiting time was a function of the arrival rate only, then an increase in customer arrivals would cause an increase in the waiting time. This is not the case as can be seen from Figures 7 and 8.

The waiting time distributions for both days were determined and the results are shown in Figures 9 and 10. The waiting time as observed deviated from a first-come, first-serve sales policy. To see the effect of a true first-come, first-serve policy, the waiting time of each customer was recomputed with the servicing time the same as in the original observation. These results are also given by Figures 9 and 10. On both days, if the sales policy had been a true first-come, first-serve, the customers would have waited longer for service.

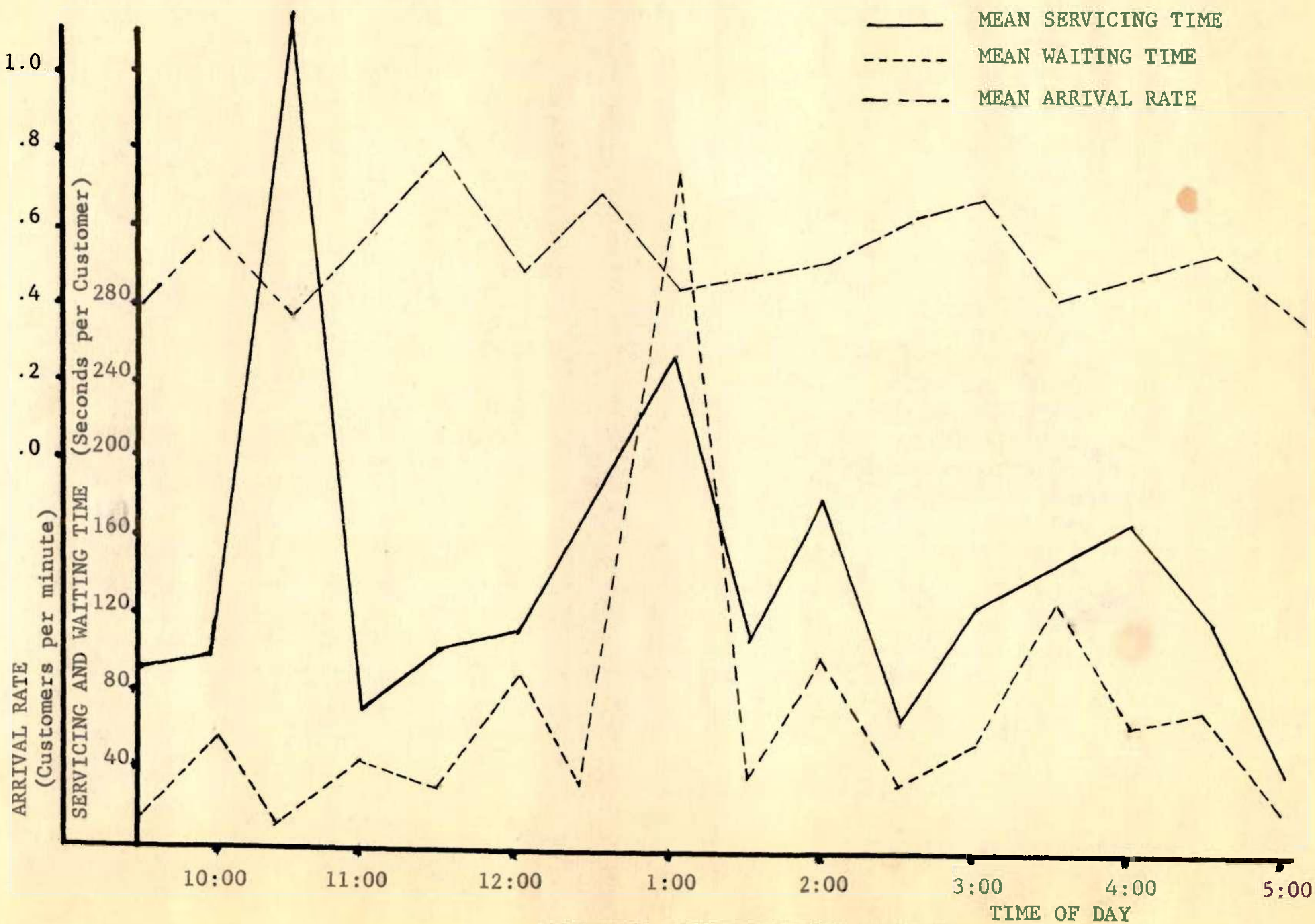

FIGURE 7 TIME CHART FOR TUESDAY

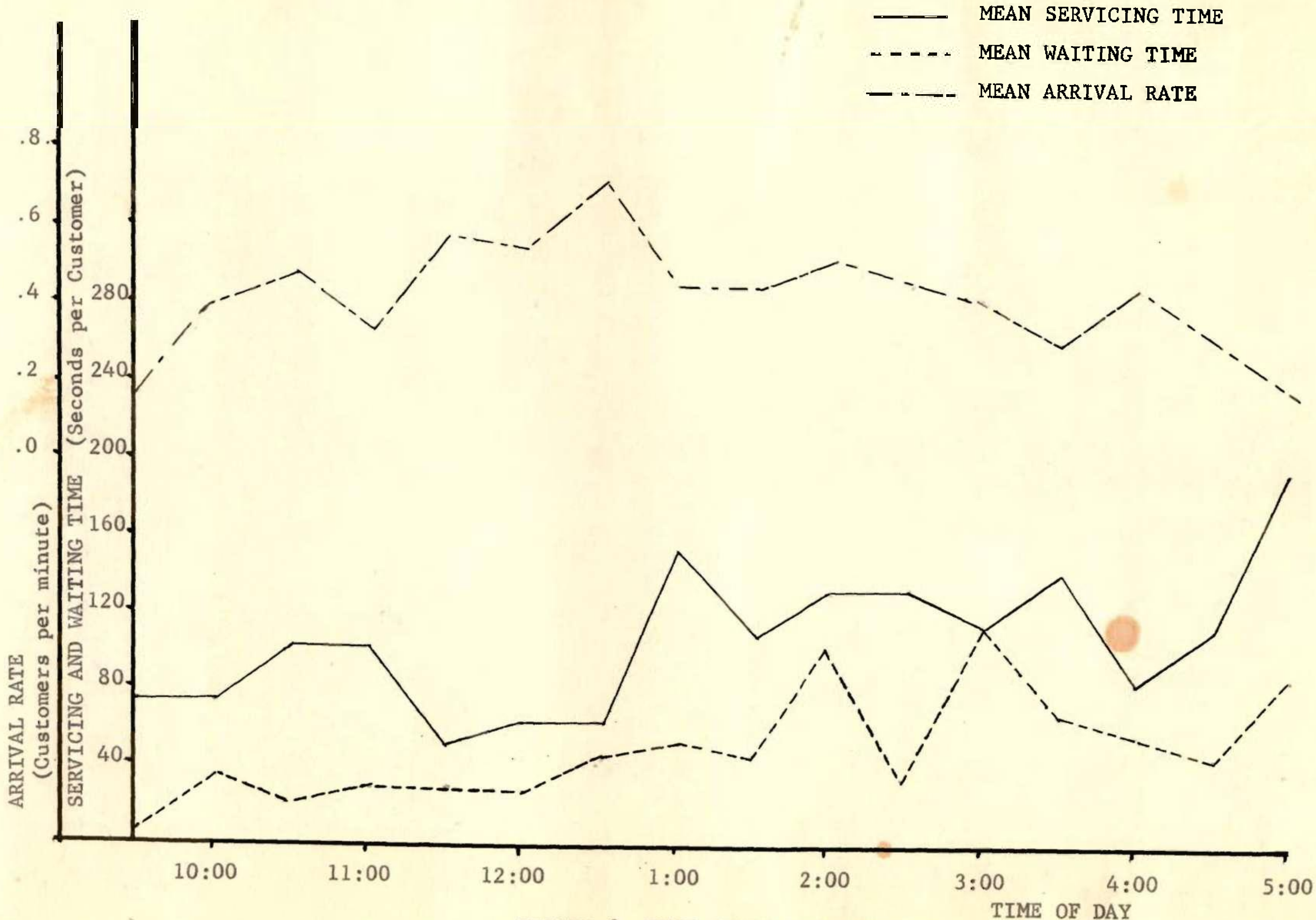

FIGURE 8 TIME CHART FOR SATURDAY

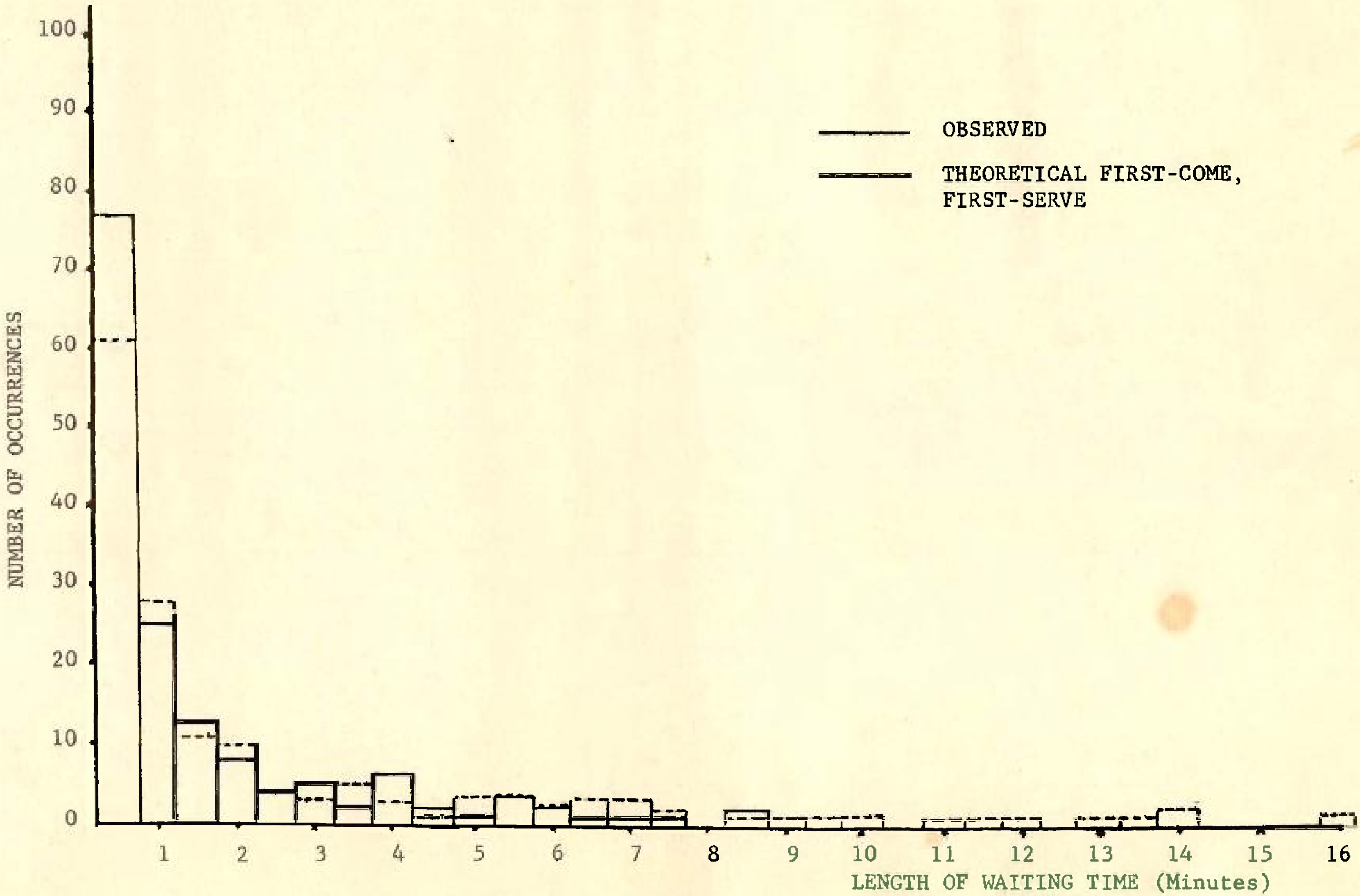

FIGURE 9 WAITING TIME DISTRIBUTION FOR TUESDAY

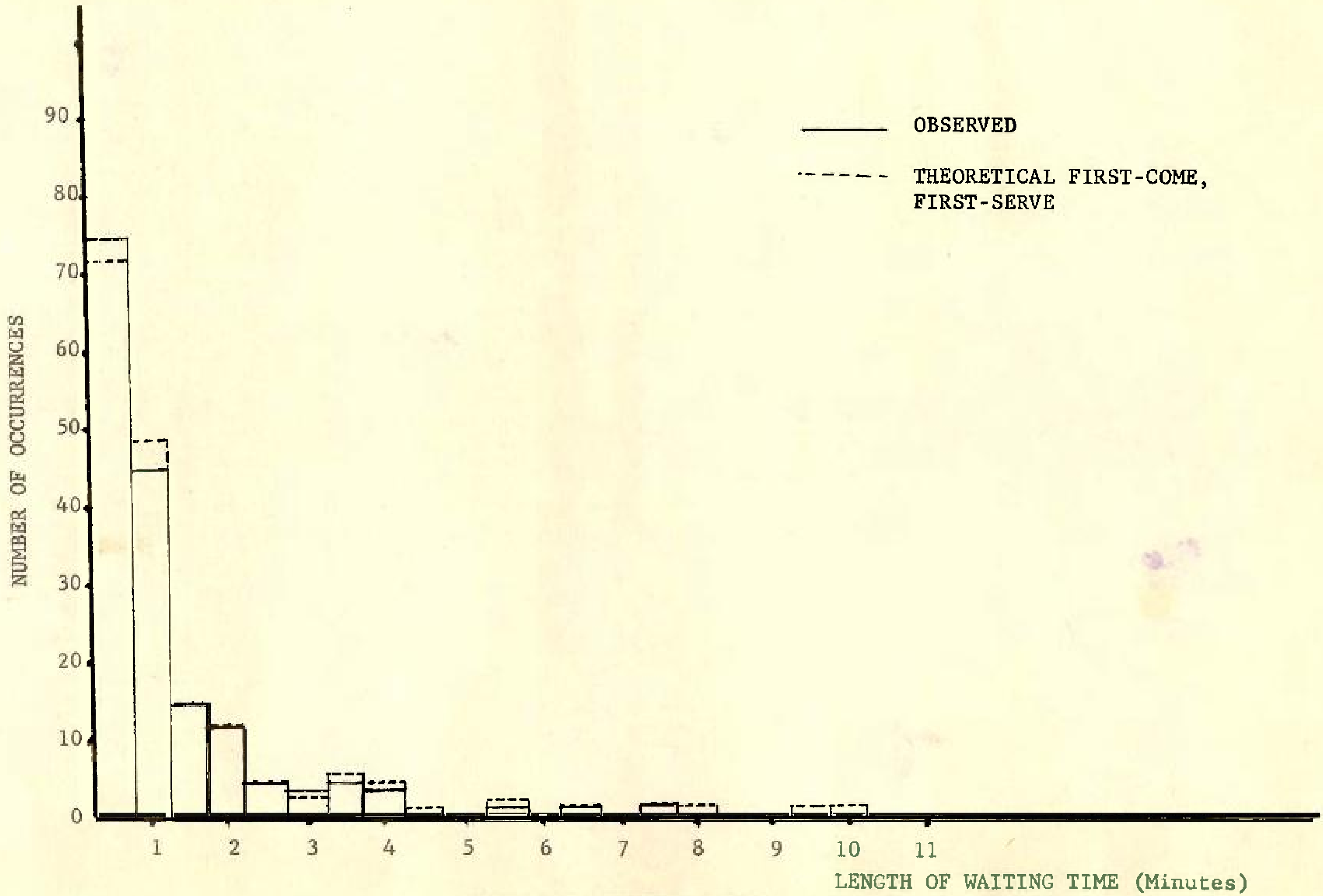

FIGURE 10 WAITING TIME DISTRIBUTION FOR SATURDAY

A closer look at the waiting time shows the extreme difference of the days on which the data were collected. Figures 11 and 12 which show the time of day versus the average waiting time of customers grouped serially in fours, clearly point out the difference between the days. On Saturday, the length of the average waiting time increases gradually until four o'clock when it tapers off. Tuesday's waiting time is quite different and is greatly influenced by the customers who are housewives and those who are employed. Around the lunch hour the effect of the workers was particularly noticed.

Also from Figures 11 and 12, the total length of time taken by a group of four shows little effect on the average waiting time, nor is the total length of time taken by the group affected by the time of day.

An Economic Application.--In an attempt to minimize the length of the waiting line and at the same time minimize the clerk's idle time, a compromise might be brought about by finding the optimum economic relationship between the cost of the sales clerk and the value of customer's time. The following is put in the form of a suggestion, as more study of the application of the queuing theory to retail stores would be required before the utility of this approach could be verified. This economic approach will consider both the single channel (one sales clerk) and the multi-channel counter.

Single Channel.--From the results of Gresham's (17) work, the following conclusions were drawn:

1. The arrival rate is random and constant

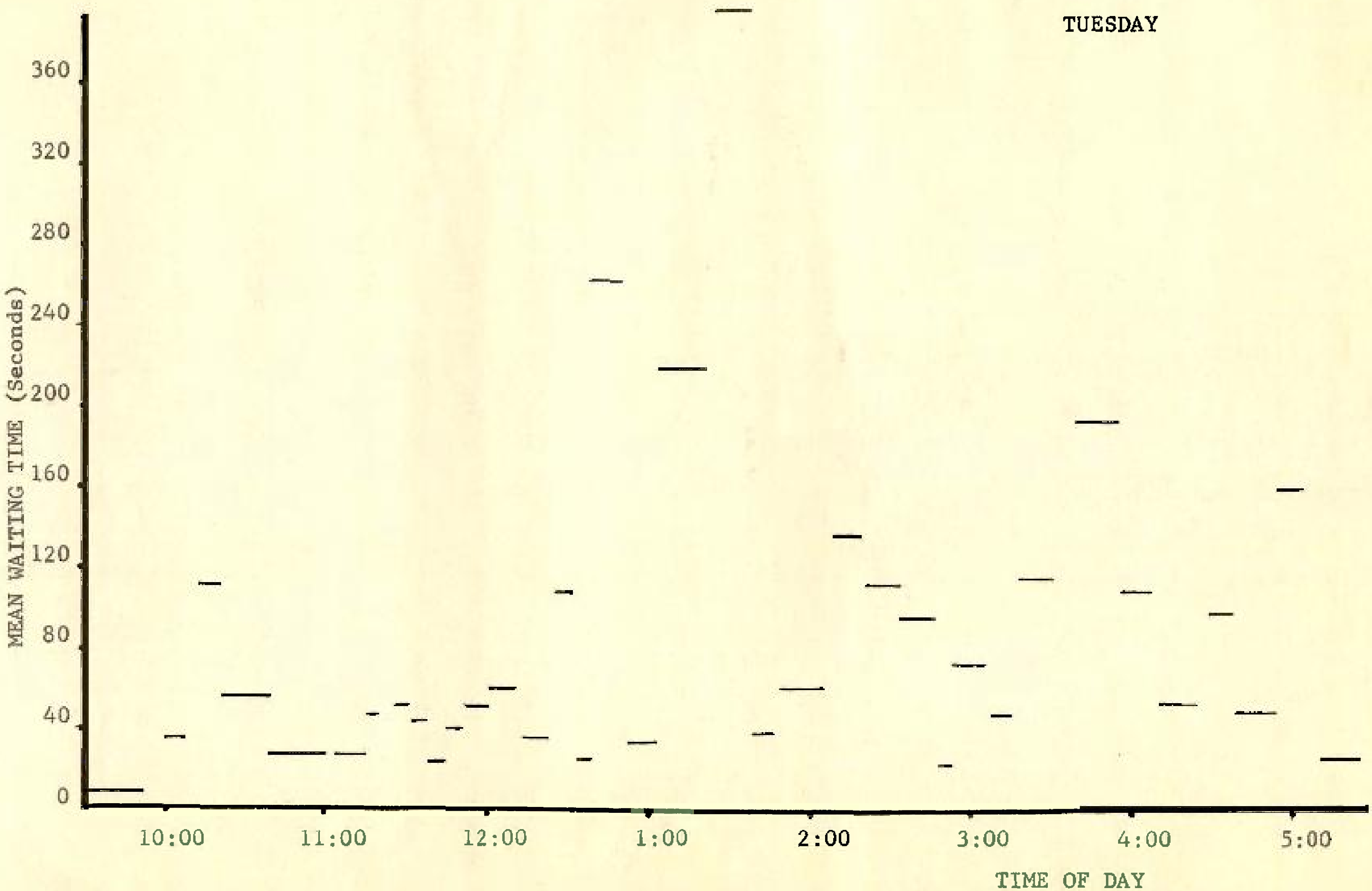

FIGURE 11 MEAN WAITING TIME FOR CUSTOMERS GROUPED IN FOURS

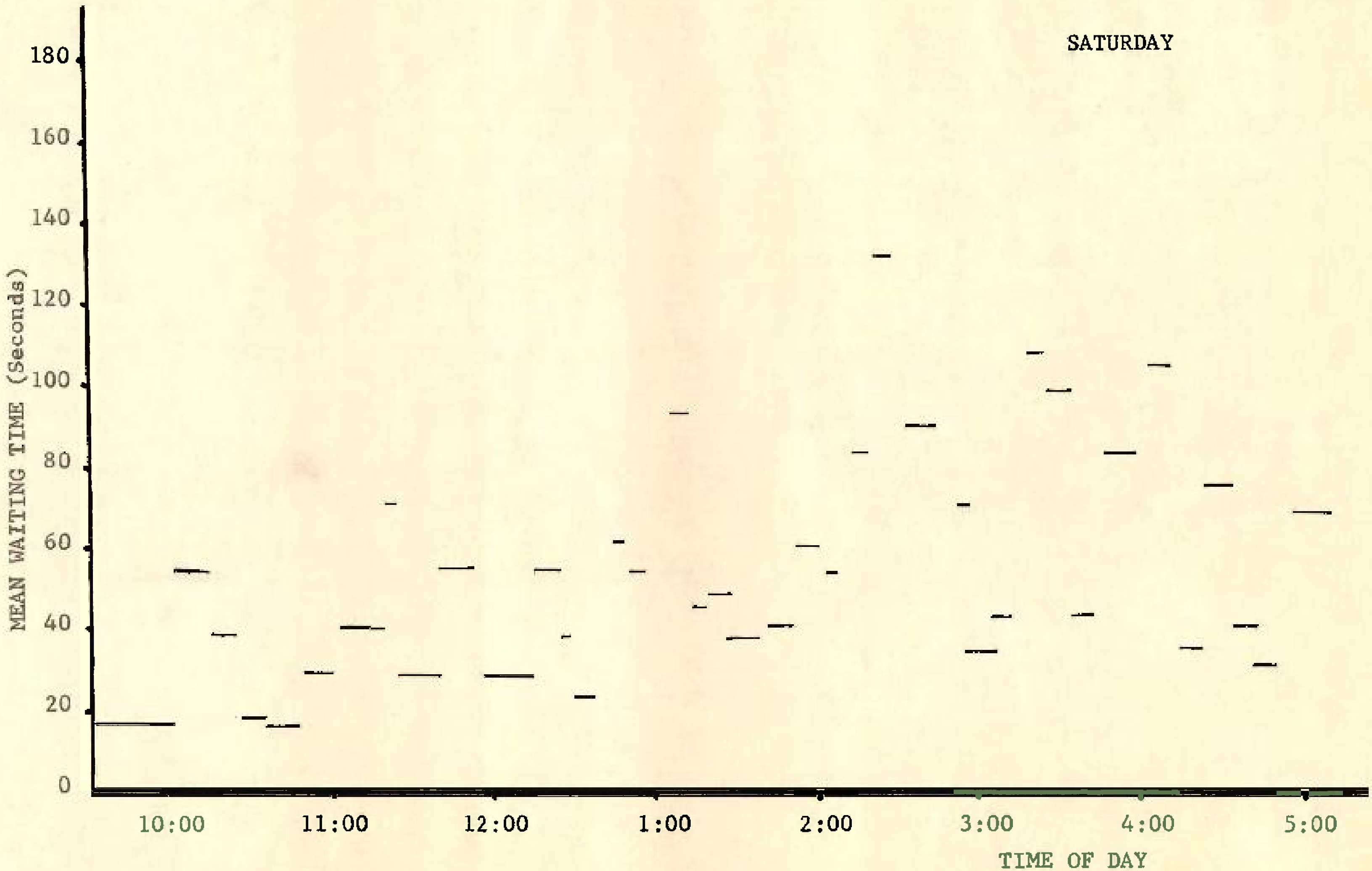

FIGURE 12 MEAN WAITING TIME FOR CUSTOMERS GROUPED IN FOURS

2. The service time distribution can be characterized by the mean and other moments.

To start an analysis by the use of "J" curves, data must be obtained so that the mean arrival rate, the mean service rate, and the variance of the servicing time can be calculated. From these calculations, the expected total time of a customer at a counter can be obtained from the following expressions by Churchman (18):

$$\bar{t}_t = \bar{t}_w + \bar{t}_s = \frac{1}{\lambda}\left[\frac{\lambda}{\mu} + \frac{\lambda^2 \sigma_{ts}^2 + \left(\frac{\lambda}{\mu}\right)^2}{2\left[1 - \frac{\lambda}{\mu}\right]}\right] \quad (1)$$

where $\bar{t}_w$ = expected waiting time (minutes per customer)

$\bar{t}_s$ = expected servicing time (minutes per customer)

$\bar{t}_t$ = expected total time (minutes per customer)

λ = mean arrival rate (customers per minute)

μ = mean service rate (customers per minute)

$\sigma_{\bar{t}_s}^2$ = variance of servicing time (minutes)

Thus for a given variance of servicing time, the expected total time a customer will stay at a counter can be expressed in terms of λ and μ. This enables one to assemble tables for the expected total time for different variances in terms of λ and μ. The total time all the customers spend at the counter is

$$W_c = N\left(\bar{t}_w + \bar{t}_s\right) \quad (2)$$

where N = number of customers

W_c = total time of all customers at counter (minutes)

The sales clerk is available for a total time, T, or all the sales clerks are available, sT, where s is the number of clerks. Since this is single channel, the total time of the sales clerk is

$$W_s = T \qquad (3)$$

where W_s=total time of sales clerk (minutes)

By letting R_s be equal to the cost of the sales clerks (dollars per unit of time), including any commission earned, and R_c be the value of the customers' time as appraised by the management of the store, the total cost of the sales clerks and customers' time at the counter can be expressed as:

$$C = R_c W_c + R_s W_s \qquad (4)$$

By substituting in Equation 4 those values in Equations 2 and 3, the following will result:

$$C = R_c N \left(\bar{t}_w + \bar{t}_s\right) + R_s T \qquad (5)$$

and can be rearranged as

$$\frac{C}{R_s T} = \frac{R_c}{R_s} \frac{N}{T}\left(\bar{t}_w + \bar{t}_s\right) + 1 \qquad (6)$$

Since C is the total cost and R_sT is the cost of one sales clerk per unit of time, the total cost per cost of one clerk for one day is $\frac{C}{R_sT}$

which can be called the normalized cost and denoted as C^1. The normalized cost can be measured in units equal to the cost of one clerk per unit of time. Rewriting Equation 6, the following is obtained:

$$C^1 = \frac{R_c}{R_s} \frac{N}{T} \left(\bar{t}_w + \bar{t}_s \right) + 1$$

Since $\frac{N}{T}$ is the average number of arrivals per unit of time or λ; and if the ratio, $\frac{R_c}{R_s}$, is defined as R ; then Equation 7 can be further reduced to

$$C^1 = R \, \lambda \left(\bar{t}_w + \bar{t}_s \right) + 1 \qquad (8)$$

Then by obtaining the value for the total expected time, $\bar{t}_w + \bar{t}_s$, in Equation 1 for a specified variance, σt_s^2 . C^1 can be expressed in terms of three variables, λ , μ, and R . These calculations can be made for all values of variances; thus a table will be formed with C^1 being expressed for each value of a variance. From this data "J" curves can be plotted with the arrival rate for a given mean service time versus the normalized costs for different values of the ratio, R.

An application of this approach is given for Tuesday's data obtained from Gresham's (17) work. The following are the values he obtained:

1. λ = 0.358 (customers per minute)
2. μ = 0.501 (customers per minute)
3. σt_s^2 = 4.32

This type of data can be obtained very easily as done in Gresham's (17) thesis, and from this a table such as Table 5 can be calculated. This table gives the customer's expected total time at the counter in terms of λ and μ for a given value of σt_s^2, of in the example, the variance 4.32. Once the table is obtained, values for C^1 can be calculated with the use of Equation 8 by varying λ, μ, and R. Since in the example $\mu = .5$, only λ and R were varied and C^1 was calculated. C^1 was then plotted against the arrival rate for a specific σt_s^2 (4.32), and the results are shown in Figure 13.

Multi-Channel.--For a single channel Churchman (18) gives Equation 1

$$\bar{t}_t = \frac{1}{\lambda}\left[\frac{\lambda}{\mu} + \frac{\lambda^2 \sigma t_s^2 + \left(\frac{\lambda}{\mu}\right)^2}{2\left(1 - \frac{\lambda}{\mu}\right)}\right]$$

for random input and a given distribution of service with variance σt_s^2 and mean μ. This was the case for the retail store for a single channel sales point. Since the data observed were obtained from a one-channel source there is no accurate way of describing the multi-channel service distribution. However, Churchman (18) also gives an expression for a customer's expected total time for random input and output which is:

$$\bar{t}_w + \bar{t}_s = \frac{1}{\mu - \lambda} \qquad (9)$$

If $\sigma t_s^2 = \frac{1}{\mu^2}$ Equation 1 becomes Equation 9. The total expected time for both days was calculated by both Equations 1 and 9, and the variation of the total expected time was less than five per cent. Since this is true, the multi-channel service time distribution can

TABLE 5

EXPECTANT TOTAL TIME PER CUSTOMER FOR

$\bar{t}_s^{2}$ OF 4.32 MINUTES

MEAN ARRIVAL RATE, λ, (CUSTOMERS PER MINUTE)	MEAN SERVICE RATE (CUSTOMERS PER MINUTE) .1	.2	.3	.4	.5	.6	.7
	EXPECTANT TOTAL TIME (MINUTES/CUSTOMER)						
.1	---	7.8	4.5	3.2	2.5	2.0	1.7
.2	---	---	8.1	4.6	3.4	2.7	2.2
.3	---	---	---	8.8	5.3	3.8	3.0
.4	---	---	---	---	10.0	5.8	4.2
.5	---	---	---	---	---	12.0	6.7
.6	---	---	---	---	---	---	13.1

NOTE: ---HAS BEEN INSERTED WHENEVER $\frac{\lambda}{\mu} \geq 1$, IN WHICH CASE THE QUEUE GROWS BEYOND ALL BOUNDS.

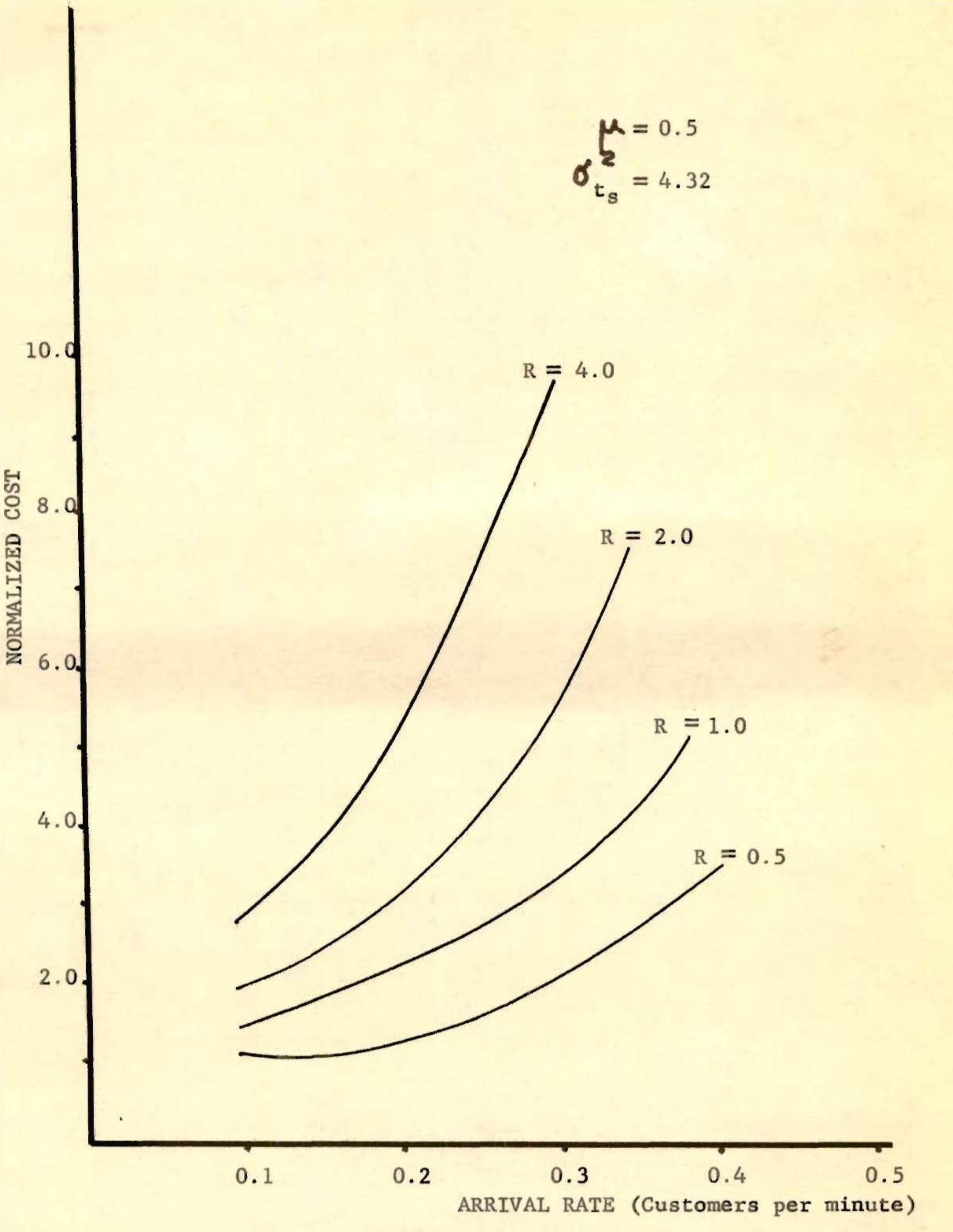

FIGURE 13 SINGLE CHANNEL "J" CURVE

be assumed to be negative exponential without introducing too much error in this application. If one assumes it is, Churchman (18) gives an expression for the expected waiting time for random arrival and exponential service for multi-channel as

$$t_w = \frac{P_o \left(\frac{\lambda}{\mu}\right)^s}{\mu(s)\ (s)!\ [1 - \lambda/\mu s]^2} \qquad (10)$$

where

$$P_o = \frac{1}{\sum_{n=o}^{S-1} \frac{(\lambda/N)^n}{NT} + \left\{\left(\frac{\lambda}{\mu}\right)^s \Big/ \left[s!\ (1 - s)\right]\right\}} \qquad (11)$$

Therefore the expected total time for the customer is

$$\bar{t}_t = \bar{t}_w + \bar{t}_s$$

or

$$t_t = \frac{P_o \left(\frac{\lambda}{\mu}\right)^s}{\mu(s)\ (s!)\ \left[1 - \lambda/\mu s\right]^2} + \frac{1}{\mu} \qquad (12)$$

By the same procedure as for the single channel, the following normalized cost equation can be written:

$$C^1 = R\,\lambda\left(\bar{t}_w + \bar{t}_s\right) + S \qquad (13)$$

With Equations 12 and 13, it is possible to plot the normalized cost versus the arrival rate for a specific mean service time at different cost ratios. This was done for Tuesday's data and the results combined with Figure 13 to give Figure 14. Thus Figure 14 gives the normalized

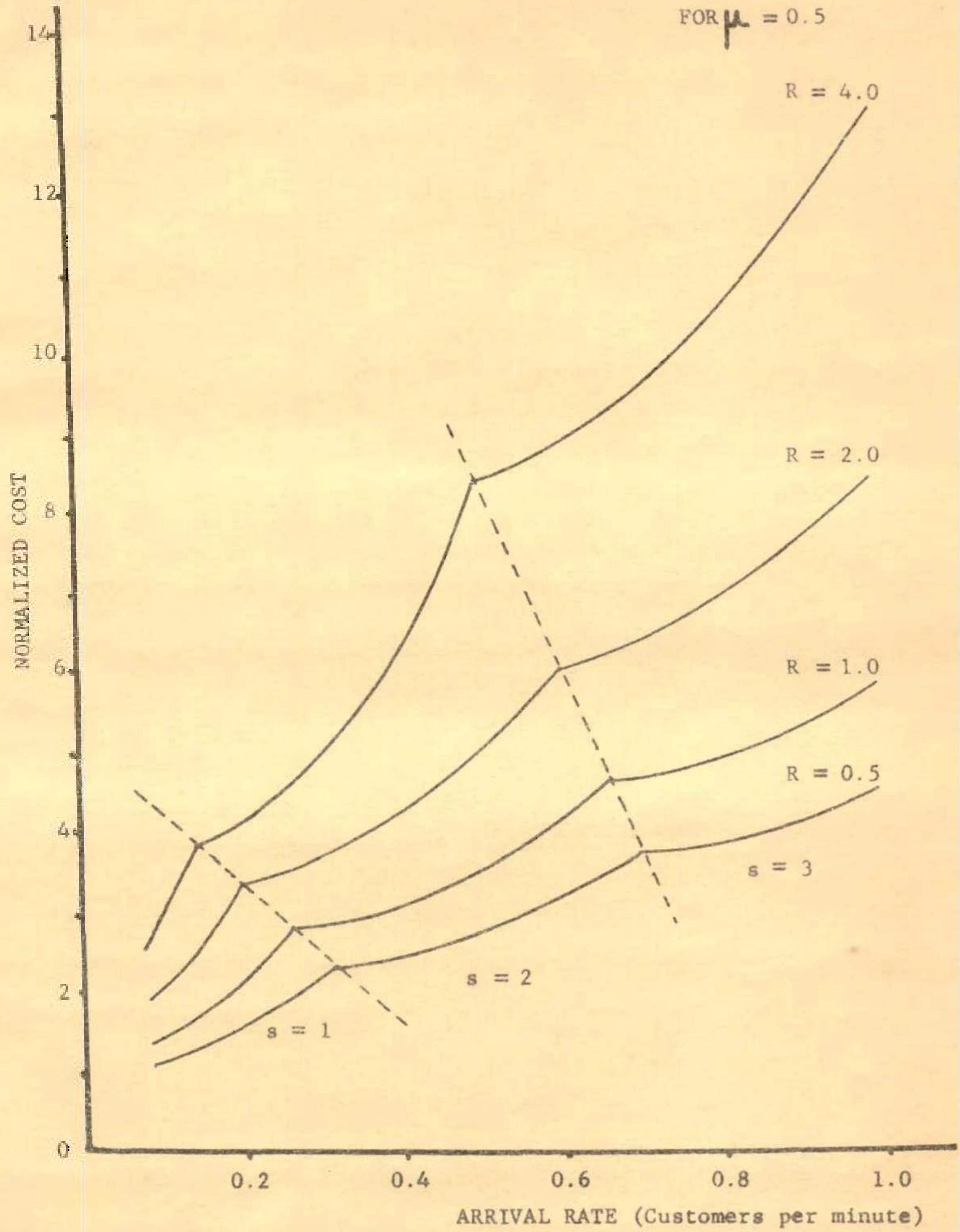

FIGURE 14 "J" CURVE FOR MULTI-CHANNEL

cost for different arrival rates, different cost ratios, and different numbers of sales clerks (all for a given mean service rate of .5). By a similar technique these curves could be drawn for all values of λ. The value of C^1 is a family of "J" curves for each number of clerks. The intersection of the "J" curve with the same ratio, R, would be the most economical point to add or reduce the sales force.

The application of this suggested "J" curve approach depends upon the way in which the management approaches the problem of optimum assignment of sales clerks. The value of the customer's time, R_c, is a decision that has to be made by management: therefore, the applicability of the "J" curve approach depends on the ability of management to assign intelligently a value to R_c. It should be a value with which management would strive for either short range or long range optimum profit. If it is decided that long range profit is to be stressed, the value of R_c would be larger than a short range profit was desired.

Once the value of the customer's time has been determined, the application of the "J" curves to a retail sales point would be simple. Data from which the arrival rate for different hours of the day must be compiled. With the knowledge of the mean service rate, the variance of the service time, and the ratio of the value of the customer's time to the cost of the clerk; the optimum number of sales clerks can be read from the "J" curves for any given arrival rate.

The Effect of the Queue Discipline on the Mean Waiting Time.--An interesting result of this study is the comparison of the mean waiting times seen in Table 6. Here the mean waiting time for Tuesday as observed from the films was 1.33 minutes per customer, yet by Equation 1, the expected waiting time was 5.0 minutes. Since this equation was a first-come, first-serve sales policy and in the actual case there was a small deviation from such a policy, one would expect a small difference in the mean waiting times. The difference of these times, however, was too large to attribute to the sales policy along. The mean waiting time was then determined for the observed data recalculated for a true first-come, first-serve policy. This time the mean waiting time was 3.4 minutes per customer, which approaches the expected value given by Equation 1.

There are several reasons that might bring about this result. Perhaps the sales clerks have a priority service policy whereby an aggressive customer could get service quicker than other customers, or maybe the sales clerks know which customer will require the least time for service. In an attempt to find the answer to the differences in the mean waiting times, the sales clerks were interviewed. They said that it is possible sometimes to tell if a customer will require a short or long time for service, but they still followed a first-come, first-serve policy regardless of this. In the case of an aggressive customer, the clerks politely told them that they must wait their proper turn. One clue to the difference in the waiting times was given by the clerks. Since a retail store is self-service to a certain extent, the clerk would serve a customer and while the customer was deciding on

TABLE 6

MEAN CUSTOMER WAITING TIME

	TUESDAY WAITING TIME (MINUTES/CUST.)	TUESDAY %	SATURDAY WAITING TIME (MINUTES/CUST.)	SATURDAY %
OBSERVED TIMES *	1.3	26	0.95	20
FIRST-COME, FIRST-SERVE TIME **	3.4	68	1.8	38
THEORETICAL FIRST-COME, FIRST-SERVE TIME ***	5.0	100	4.8	100

NOTE: *THIS IS THE MEAN WAITING TIME AS OBTAINED FROM THE DATA

**THIS IS THE MEAN WAITING TIME FROM THE OBSERVED DATA THAT WAS RECALCULATED FOR A TRUE FIRST-COME, FIRST-SERVE SALES POLICY

***THIS IS THE EXPECTED WAITING TIME CALCULATED BY EQUATION 9.

a belt that she wanted, the clerk would serve another customer. This could tend to balance the waiting times, for the clerk was actually waiting on two people at once.

The "J" curve approach for the optimum assignment of sales clerks will definitely be affected by this difference of the mean waiting times. The curves as shown in Figures 13 and 14 are not a true picture of the retail sales counter. This is not a major defect, for the "J" curves were only an attempt to show an approach to the assignment of sales personnel. With the proper adjustment to take into account the self-service feature of a retail sales point, the "J" curves could be obtained in a similar manner as done in this thesis.

The difference of the calculated expected waiting time from the observed waiting lines means that the mathematical model will have to have a new approach in order to fit the self-service feature. Basically the arrival rate and the servicing time at a retail sales point can be characterized. All that remains to be done before a mathematical model can be determined is to characterize the type of queue discipline and to develop an analytical solution of such a model. Once this has been done, the "J" curves can be used effectively.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions.--The conclusions that can be drawn from this study are as follows:

1. The results of the personal interviews, which were approximately the same for both days, show that more than ninety-six per cent of the customers leave the queue for reasons other than the length of the waiting line. The loss of the four per cent potential sales does not justify the addition of more sales personnel.
2. There is a departure from a first-come, first-serve sales policy. As the length of the queue increases so does the variation in the order of service. This departure from a first-come, first-serve sales policy is expected because of the limitations of the sales clerks to remember the proper order sequence during a crowded period. The magnitude of the deviation is small. If x equals the size of the queue when served and y equals the deviation in the order of service, then the equation for the mean deviation from a first-come, first-serve sales policy may be written as follows:

$$y = .12 + .14x$$

This equation is the average of the equations given on Page 19.

3. The mathematical model of a retail sales point requires a different approach due to the effect of the queue discipline on the expected waiting line.

Recommendations.--In view of the limitations, results, and conclusions of this study, the following recommendations are made:

1. It is felt that further study of this problem would be profitable. Future investigations should be pointed in the following directions:
   a. an investigation to see if the queue discipline can be characterized so that the application of the queuing theory to a retail sales point can be made.
   b. a study to determine if more sales personnel would attract the customers that looked without being serviced.
   c. a further investigation to determine the validity of the "J" curve approach with particular reference to the service time distribution assumptions required for multi-channel mathematical models.
2. Once a model is determined for a retail sales point, an evaluation of it and all types of counters in a department store should be made.

A P P E N D I X

PERSONAL INTERVIEW

DATE________________ TIME______________

1. Were you:

____ (a) definitely planning to buy a belt

____ (b) going to buy a belt if you saw one that appealed to you

____ (c) just brousing and perhaps might buy a belt

____ (d) looking without any intention of buying

CONTINUE INTERVIEW IF ANSWER IS NOT 1 (d) ABOVE.

2. Why did you leave the counter:

____ (a) too many people waiting

____ (b) did not have size belt that I wanted

____ (c) did not have style belt that I wanted

____ (d) did not have color belt that I wanted

____ (e) other reasons____

____

CONTINUE INTERVIEW IF ANSWER IS 2 (a) ABOVE

3. Do you plan to return to this counter today?

____ yes

____ no

4. Are the belts in your price range:

(a) ____ \$1-3 ____ \$3-5 ____ \$5 and over

____ (b) did not notice price

____ (c) low

____ (d) high

FIGURE 15 SAMPLE INTERVIEW FORM

ARRIVAL TIME	DEPARTURE TIME	TOTAL TIME	SERVICE START	SERVICE END	SERVICE TIME	WAITING TIME	SALE OR NO SALE	ORDER SERVED	REMARKS
4:07:39	4:11:01	3:22	4:07:49	4:11:01	3:12	:10	1-D	0	
4:07:49	4:08:16	:27							
4:08:18	4:09:08	:50							
4:08:37	4:10:34	1:57							
4:11:04	4:14:37	3:33	4:11:54	4:14:37	2:43	:50	S	0	
4:11:06	4:11:49	:43							
4:11:16	4:12:20	1:04							
4:11:54	4:13:25	1:31							
4:11:56	4:12:06	:10							

FIGURE 16 SAMPLE DATA SHEET

TABLE 7

AVERAGE ARRIVAL, WAITING, AND SERVICING

TIMES FOR TUESDAY

BEGINNING TIME	ARRIVAL (CUST./MIN.)	WAITING (SEC./CUST.)	SERVICING (SEC./CUST.)
9:30	0.200	5.0	94.3
10:00	0.400	53.3	101.0
10:30	0.200	18.4	440.8
11:00	0.400	43.5	74.6
11:30	0.600	34.8	104.3
12:00	0.300	92.1	115.5
12:30	0.500	80.8	192.8
1:00	0.267	351.2	259.1
1:30	0.300	40.0	109.2
2:00	0.333	102.4	185.3
2:30	0.435	39.1	68.1
3:00	0.500	59.1	128.5
3:30	0.234	132.8	147.2
4:00	0.300	67.2	174.2
4:30	0.368	79.9	127.2
5:00	0.231	29.8	44.7

TABLE 8

AVERAGE ARRIVAL, SERVICE, AND WAITING TIME

FOR SATURDAY

BEGINNING TIME	ARRIVAL (CUST./MIN.)	WAITING (SEC./CUST.)	SERVICING (SEC./CUST.)
9:30	0.167	10.4	78.7
10:00	0.400	37.5	74.0
10:30	0.477	22.2	101.6
11:00	0.333	29.7	103.3
11:30	0.571	31.2	55.5
12:00	0.535	26.5	63.3
12:30	0.705	41.5	63.9
1:00	0.435	49.3	150.5
1:30	0.435	45.5	108.2
2:00	0.500	101.9	132.4
2:30	0.435	37.4	132.3
3:00	0.400	117.7	113.0
3:30	0.300	66.1	142.3
4:00	0.435	57.3	80.7
4:30	0.300	43.4	113.1
5:00	0.167	82.6	193.2

TABLE 9

CODE OF THE INTERVIEW FORM

THE CODE WAS MADE BY A NUMBER SIGNIFING WHY A CUSTOMER APPROACHED THE COUNTER AND A LETTER TELLING WHY SHE LEFT WITHOUT PURCHASING. THE COMPLETE CODE IS GIVEN BELOW:

1. DEFINITELY GOING TO PURCHASE A BELT BUT
   - A. TOO MANY PEOPLE WAITING
   - B. DID NOT HAVE THE SIZE BELT THAT THEY WANTED
   - C. DID NOT HAVE STYLE BELT THAT THEY WANTED
   - D. DID NOT HAVE COLOR BELT THAT THEY WANTED
   - E. OTHER REASONS

2. GOING TO BUY A BELT IF THEY SAW ONE THAT APPEALED BUT
   - A. TOO MANY PEOPLE WAITING
   - B. DID NOT HAVE SIZE BELT THAT THEY WANTED
   - C. DID NOT HAVE STYLE BELT THAT THEY WANTED
   - D. DID NOT HAVE COLOR BELT THAT THEY WANTED
   - E. OTHER REASONS

3. WERE BROWSING AND PERHAPS MIGHT BUY A BELT BUT
   - A. TOO MANY PEOPLE WAITING
   - B. DID NOT HAVE THE SIZE BELT THAT THEY WANTED
   - C. DID NOT HAVE THE STYLE BELT THAT THEY WANTED
   - D. DID NOT HAVE THE COLOR BELT THAT THEY WANTED
   - E. OTHER REASONS

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